

1. INTRODUCTION TO BRIDG-IT

1.1. History.

- Bridg-it was created by David Gale, a mathematician, around 1960.
- It was first published in *Scientific American* as the Game of Gale.

1.2. How to play Bridg-It.

- The game board is two rectangular arrays of dots, one array is for player one and the other for player two, where each interior dot of player 2's array lies inside a one by one square formed from player 1's array.
- The players alternately connect two adjacent dots of their own color with a bridge.
- A bridge cannot cross a bridge already played.
- A player wins when they connect the opposite edges of the board that are their color.

1.3. Question: What can you do to win the game?

2. INTRODUCTION TO GRAPH THEORY

2.1. Definitions.

2.1.1. *Graph.* A graph G is a set of points V (called vertices) together with connecting lines E (called edges).

2.1.2. *Subgraph.* A subgraph H is a graph where all the vertices of H are also vertices of G , and every edge in H is also an edge in G .

2.1.3. *Path.* A path in a graph is an alternating sequence of vertices and edges, beginning and ending at a vertex, where each vertex is incident to the edges that precede and follow it in the sequence.

2.1.4. *Cycle.* A cycle is a path where the beginning and end are the same vertex.

2.1.5. *Connected.* A graph is connected if there is a path between any two vertices.

2.2. **Question:** If you have n vertices, what is the minimum number of edges you need to create a connected graph?

2.3. **Question:** Describe the properties of the graphs on n vertices you found above:

- (1) number of edges
- (2) number of shortest paths between two specified vertices x and y
- (3) number of cycles

3. APPLICATION

Let the communities in south-east Nebraska and a large power company be the vertices of a graph and an edge will represent power lines connecting the towns to each other and to the generator. Neglecting the length of the power lines, what is the minimum number of power lines needed so that every community can receive power? Such a graph is called a spanning tree.

3.1. What happens if one power line is brought down, say by ice, and that one line cannot be repaired? How many new power lines need to be added to restore power to all communities?

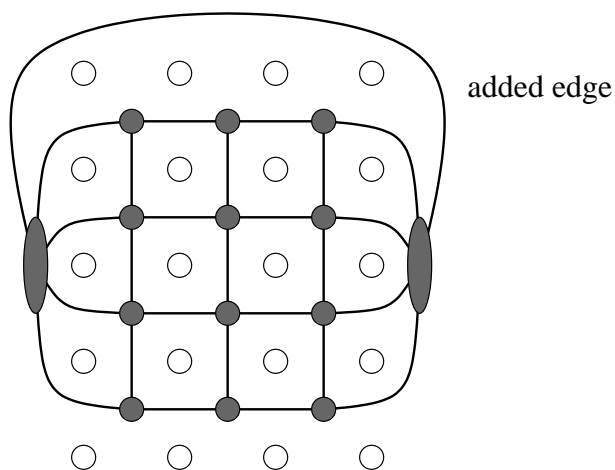
3.2. The power company is unsatisfied with its service and so builds more power lines. The resulting graph of power lines consists of *two* edge-disjoint spanning trees, one of which is used as the primary system and the other is held in reserve.

3.2.1. Draw such a graph.

3.2.2. *Question:* Suppose a power line fails in the primary spanning tree. What can the company do to restore power to all communities? Does the company have to use all of the power lines of the reserve spanning tree? What is the minimum number of power lines in the reserve spanning tree that the company needs to use?

4. PLAYING BRIDG-IT USING GRAPH THEORY

4.1. **What is the graph?** We have a graph for Player 1, with sides combined into single vertices. Each time Player 2 makes a move, they are actually erasing an edge from the graph. Each time Player 1 makes a move, they are coloring an edge, making it off limits for Player 2 to erase.



4.2. **Player 2 takes the first turn.** Player 2 takes the added edge first since it didn't belong in the game initially.

4.3. **Question:** What should Player 1 do on each turn?

5. SIMILAR GAMES

5.1. Shannon Switching Game.

5.1.1. *History.* This game was created around the same time as Bridg-It independently by Claude Shannon, “the father of information theory.” The main difference between this game and Bridg-It is the game board.

5.1.2. *Rules.*

- The game is played on a finite graph labeled with two special vertices A and B.
- The edges of the graph are initially all labeled 0.
- The players alternate turns. Player 1 can change the label of any edge to a 1. Player 2 can delete any edge that is labeled 0.
- Player 1 wins if she can connect A and B with a path of edges labeled 1. Otherwise Player 2 wins.

5.1.3. *Question:* Can this game ever end in a tie?

5.1.4. *Question:* Is there a winning strategy for either player?

5.2. HEX.

5.2.1. *History.*

- This game was invented by a Danish mathematician Piet Hein in 1942, and also independently invented by the mathematician John Nash in 1947.
- Like the games listed above, this game is in the category of connection games.
- It is said that Nash invented HEX in response to the ancient Asian game of Go.

5.2.2. *Rules.*

- The game is played on a hexagonal grid in the shape of a rhombus.
- Each player has an allocated color and the players alternate turns coloring a hexagon.
- The first player to link opposing sides of the board marked by their colors is the winner.

5.2.3. *Question:* Can this game ever end in a tie?

5.2.4. *Question:* Is there a winning strategy for either player?

